Int. J. Heat Mass Transfer. Vol. 32, No. 12, pp. 2375–2384, 1989
Printed in Great Britain

0017–9310/89 $3.00+0.00

A modelling study to the heat transfer between immersed surfaces and large-particle fluidized beds

JIDONG LU and RENZHANG QIAN

Department of Power Engineering, Huazhong University of Science and Technology, Wuhan, Hubei, People's Republic of China

(*Received* 27 *June* 1988 *and in final form* 7 *March* 1989)

Abstract—A new theoretical model is proposed for the heat transfer between immersed surfaces and large-particle fluidized beds. The heat transfer of the emulsion phase to the immersed surface is treated as the sum of the convective part of the interstitial fluid flow and the conductive part of the solid particles. The heat transfer of bubbles to the surfaces is also considered. The theoretical calculating formulae obtained are in fairly good agreement with experimental data in a large region.

INTRODUCTION

SINCE the fluidization technology was applied to coal burning in recent years, the important subjects of large particle ($d_p > 1$ mm) fluidized beds, such as the hydrodynamics of fluid and particles, the heat and mass transfer between the immersed surfaces and the bed as well as the particles and fluid, the absorption of sulphur, etc. have been studied by many workers.

The dependence of the heat transfer of the immersed surface to the fluidized bed on the physical properties of the fluid and bed material, the structure of the bed and the operation conditions was found through a lot of experimental studies, which gave the heat transfer mechanism to a certain extent. But knowledge on this basis is not enough and experimental results are different from each other because of the different experimental conditions. In order to gain an essential understanding of the flow and heat transfer in industrial equipment and direct the optimum design and operation, it is necessary to carry out the mechanismic modelling study.

The early studies of the heat transfer mechanical model all focused attention on fluidized beds with small particles as the bed material. They can be divided into two kinds—the particle packet model and the particle string model. Mickley and Fairbanks [1] were the first to propose the particle packet model which treated the heat transfer of immersed surfaces with the fluidized bed as the result of unsteady conduction of the particle packets with the bed temperature and the physical properties as those of the bed bulk at minimum fluidization. This model was improved upon and developed in refs. [2–4]. Botterill and Williams proposed the particle string model in 1963 [5]. They considered the unsteady conduction process of the heat transfer surface with a single particle which stayed on the surfaces. The model was then developed to the situation of a particle string [6–8]. These two kinds of models all treated the heat transfer of the immersed surfaces to the fluidized beds as the process of unsteady conduction. The analytical solution can be obtained with the particle packet model, but it can only be used for the long staying period of the particle packet and small particle beds. The numerical method should be used in the particle string models, which can be used for the short staying period situations. In other words, these two kinds of models are only suitable for cases of small particle beds with a negligible convective component. Large particle fluidized beds have different fluid flow and heat transfer characteristics than small particle fluidized beds. The fluid convective component plays a more important role in heat transfer. Several workers tried to study the heat transfer mechanism, and proposed some theoretical or semi-theoretical and semi-empirical models [9–13]. All these models reflected the characteristics of the large particle fluidized beds to a certain extent, but there exist some failings and the theoretical results are different from each other. The fluid convective part in models were all obtained through experiments except for Adams and Welty's model [13]. It is confusing that the thermal resistance of the convective component was taken as that of the packed bed [9]. The models of Zabrodsky *et al.* [10] and Ganzha *et al.* [11] took the bed voidage for that of the emulsion phase, thus the models could only be used for particulate fluidized beds. The model of Ganzha *et al.* [11] held the interstitial gas velocity to be a direct proportion to the superficial velocity. Adams and Welty's model [13] could be used for local analysis, but the basic heat transfer unit is far from the practical state and a numerical method was needed. On the basis of the above situation, a new theoretical model is proposed in this paper for relatively rational modelling the heat transfer mechanism and obtaining the rather simple calculating formulae for its practical design.

NOMENCLATURE

d_p	particle diameter
D_T	tube diameter
f_b	time fraction of bubble contact
f_{cd}	area fraction of conduction region
h_b	heat transfer coefficients of bubbles
h_e	heat transfer coefficients of emulsion phase
h_w	total heat transfer coefficients
k	turbulence kinetic energy
k_f	thermal conductivity of fluid
L_B^+	dimensionless length of boundary layer, L_B/d_p
l_p^+	dimensionless particle distance, l_p/d_p
Nu_b	Nusselt number of bubbles, $h_b d_p/k_f$
Nu_{bcd}	conductive Nusselt number of bubbles, $h_{bcd} d_p/k_f$
Nu_{bcv}	convective Nusselt number of bubbles, $h_{bcv} d_p/k_f$
Nu_{D_T}	Nusselt number of fluid across circular tube, hD_T/k_f
Nu_e	Nusselt number of emulsion phase, $h_e d_p/k_f$
Nu_{ecd}	conductive Nusselt number of emulsion phase, $h_{ecd} d_p/k_f$
Nu_{ecv}	convection Nusselt number of emulsion phase, $h_{ecv} d_p/k_f$
P	pressure
Pr	Prandtl number, $\nu_f \rho_f c_{pf}/k_f$
q	heat flux density
R^+	dimensionless radius
Re	Reynolds number, ud_p/ν_f
Re_{D_T}	Reynolds number of bubble contacting, $3U_{mf}D_T/\nu_f$
Re_w	Reynolds number near immersed tubes, $\bar{u}_w d_p/\nu_f$
r_{cd}^+	dimensionless maximum radius of conduction region, r_{cd}/d_p
T_B	bed temperature
T_w	tube surface temperature
Tu	turbulence intensity, $\sqrt{(\overline{u'^2})}/u$
U	superficial fluidizing velocity
$\bar{u}$	interstitial velocity in emulsion phase, U/ε_e
u^+	dimensionless velocity, $u/\bar{u}$
U_{mf}	superficial minimum fluidizing velocity
U_w	superficial velocity near immersed tube
v^+	dimensionless velocity, $v/\bar{u}$
x^+	dimensionless coordinate, x/d_p
y^+	dimensionless coordinate, y/d_p.

Greek symbols

γ	ratio of two boundary layer thicknesses, δ_θ/δ_v
Δ	dimensionless thickness of gas gap in conduction regions
Δ_1	maximum thickness in conduction region
δ_v	thickness of momentum boundary layer in $x\xi$ coordinate
δ_{vy}	thickness of momentum boundary layer in xy coordinate
δ_θ	thickness of temperature boundary layer in $x\xi$ coordinate
ε	voidage
ε_b	voidage in bubble phase
ε_e	voidage in emulsion phase
ε_{mf}	voidage at minimum fluidization
ε_t	dimensionless turbulent viscosity, ν_t/ν_f
$\varepsilon_{t\infty}$	dimensionless turbulent viscosity in main stream
ε_w	voidage near immersed tube
η_v	similarity variable in momentum boundary layer, ξ/δ_v
η_θ	similarity variable in temperature boundary layer, ξ/δ_θ
θ	dimensionless temperature, $(T-T_w)/(T_B-T_w)$
μ_f	fluid kinetic viscosity
ν_f	fluid kinematic viscosity
ν_t	turbulent viscosity
ρ_f	fluid density
ρ_p	particle density
ϕ_s	sphericity.

MECHANISMIC ANALYSES AND BASIC HYPOTHESES

The proposition of the mechanismic model is based on the analyses of the experimental results. The following conclusions can be made and the basic hypotheses of the physical and mathematical model can be proposed from the experimental results.

The immersed surfaces are touched by an emulsion phase and bubbles with certain voidages in a random form, which seems more regular pattern on a statistical meaning.

Hypothesis 1

The heat transfer of the immersed surfaces with the fluidized bed can be seen as the weighted average of these when the surfaces contact with the emulsion phase and the bubbles, respectively, i.e.

$$h_w = (1-f_b)h_e + f_b h_b. \quad (1)$$

The material of the fluidized beds being studied are large particles and belong to the kind of D according to the classification of Geldart [14]. The rising velocity of the bubbles is lower than the interstitial fluid velocity in the emulsion phase, and the inertia of the particle is larger. So the effect of the rising bubbles on the movement of particles is decreased and the mixing of the particles is in a lower extent compared with the small particle beds.

Hypothesis 2

The fluid–solid two-phase flow can be treated as quasi-steady. This assumption enables the flow field and pressure field to be described by the steady equations approximately.

According to the measurement of the staying period of the emulsion packet on the horizontal tubes, it is shorter than 1 s in normal operation [15]. If a qualitative analysis is carried out with the particle packet model, the heat permeating depth is 1.17 mm for a 1 s staying period of the emulsion phase which consisted of air and glass beads.

Hypothesis 3

The heat permeating depth is not greater than the first particle layer for large particle beds ($d_p > 1$ mm) at the normal fluidization state. So the particle packet model could not be used. The particles should be considered as separated and the attention is only focused on the first layer adjacent to the heat transfer surface.

It can be found from the experimental results [15] that the heat transfer coefficients in large particle beds do not change obviously with fluidizing velocity at normal operation state and the heat transfer cannot be neglected when the immersed surfaces are touched by bubbles. So it can be concluded that fluid convection plays a more and more important role with increasing particle dimensions. The fluid flow among the interstices of the emulsion phase is in a turbulent state because the eddies and wakes introduced by the flow across the particles and the turbulence energy are dissipated by the effect of fluid viscosity at the immersed surfaces. When the fluid flows around particles, impacting, separating and wake will arise, and each flow tunnel will be mixed. Such a situation we are interested in is presented near the surfaces. It is difficult to describe the fluid flow and heat transfer in detail at present. So certain simplifications must be made. The fluid flow in the interstices can be considered as the period channel flow around the particles and the flow in each period is simplified to the undeveloped boundary layer along the heat transfer surfaces if the approximation is made. Since the length of the boundary layer along the flow direction is rather small and the interstitial fluid velocity is not large, it can be thought of as the laminar boundary layer. The influence of the turbulence in the interstitial fluid flow can be considered as that of the turbulent intensity in the main stream on the boundary layer. On this basis the following hypothesis can be proposed.

Hypothesis 4

The convection component of the interstitial fluid flow in the emulsion phase can be approximated as the heat transfer of the boundary layer influenced by the turbulence in the main stream. This assumption made the analysis of the solution of this component possible.

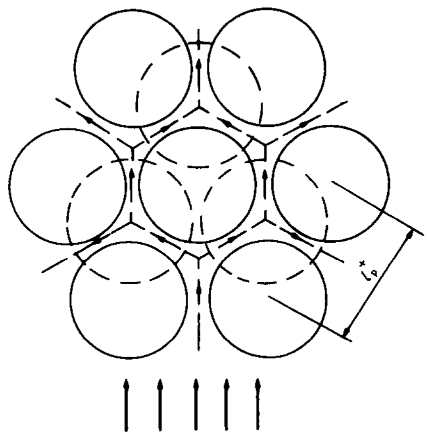

FIG. 1. Heat transfer unit.

MATHEMATICAL MODELLING OF THE EMULSION PART

In order to realize the mathematical model, some additional assumptions for the approximate treatment should be made on the basis of the above basic hypotheses.

(1) The temperature of the bed bulk T_B and the temperature of the heat transfer surfaces T_w do not change during the whole procedure of heat transfer.

(2) The bed material is considered to consist of spherical particles with equivalent diameters.

(3) The particles are assumed to be arranged in a hexahedron, as shown in Fig. 1. So the distance between the particles in the emulsion phase is $l_p = 0.9047(1-\varepsilon_e)^{-1/3}d_p$. The voidage near the heat transfer surface would be increased and in the range of half particle diameter from the surface $\varepsilon_w = 1-0.7382(1-\varepsilon_e)^{2/3}$.

(4) The interstitial fluid flow is assumed as a two-dimensional laminar boundary layer with $\bar{u} = U_w/\varepsilon_w$ as the main stream velocity. The changes of the velocity and pressure along the flow path are omitted. The length of the boundary layer is taken as $\sqrt{3}l_p$.

(5) The heat transfer of the emulsion phase to the immersed surface is divided into two regions, i.e. the conduction region adjacent to the contact points of the particles on the heat transfer surface and the convection region, as shown in Fig. 2. This is because of the interstitial fluid velocity adjacent to the contact points is very small, consequently, the convective part of the fluid can be omitted and the heat transfer is only due to the conduction perpendicular to the surfaces. At the edges of these two regions it must be satisfied by

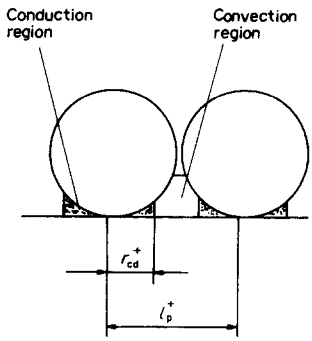

FIG. 2. Division of heat transfer regions.

$$h_{\text{ecd}} = \bar{h}_{\text{ecr}}. \tag{2}$$

Solution of the boundary layer equations

According to the basic and additional hypotheses, the convective component of the interstitial fluid flow can be described by the following dimensionless governing equations:

$$\frac{\partial u^+}{\partial x^+} + \frac{1}{1+\varepsilon_t}\frac{\partial v^+}{\partial \xi} = 0 \tag{3}$$

$$u^+\frac{\partial u^+}{\partial x^+} + \frac{v^+}{1+\varepsilon_t}\frac{\partial u^+}{\partial \xi} = \frac{1}{Re}\frac{1}{1+\varepsilon_t}\frac{\partial^2 u}{\partial \xi^2} \tag{4}$$

$$u^+\frac{\partial \theta}{\partial x^+} + \frac{v^+}{1+\varepsilon_t}\frac{\partial \theta^+}{\partial \xi} = \frac{1}{Re\,Pr}\frac{1}{1+\varepsilon_t}\frac{\partial^2 \theta}{\partial \xi^2}. \tag{5}$$

Boundary conditions are

$$\xi = 0, \quad u^+ = 0, \quad v^+ = 0 \tag{6}$$

$$\theta = 0 \tag{7}$$

$$\varepsilon_t = 0 \tag{8}$$

$$\frac{\partial^2 u^+}{\partial \xi^2} = 0 \tag{9}$$

$$\frac{\partial^2 \theta}{\partial \xi^2} = 0 \tag{10}$$

$$\xi \to \infty, \quad u^+ = 1 \tag{11}$$

$$v^+ = 0 \tag{12}$$

$$\theta = 1 \tag{13}$$

$$\varepsilon_t = \varepsilon_{t\infty} \tag{14}$$

$$\frac{\partial u^+}{\partial \xi} = \frac{\partial^2 u^+}{\partial \xi} = \cdots = \frac{\partial^n u}{\partial \xi^n} = 0 \tag{15}$$

$$\frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial \xi} = \cdots = \frac{\partial^n \theta}{\partial \xi^n} = 0. \tag{16}$$

In order to obtain the analytical solution with the integration method, the following coordinate transformation is made in the above equations and boundary conditions, which is the same as that of Adams and Welty [11]

$$d\xi = \frac{1}{1+\varepsilon_t}dy^+. \tag{17}$$

Integrating equations (4) and (5) from the heat transfer surface to the outer edge of the boundary layers δ_v and δ_θ, v^+ is removed from the continuity equations, so

$$\frac{\partial}{\partial x^+}\int_0^{\delta_v}(1-u^+)u^+(1+\xi_t)\,d\xi = \frac{1}{Re}\frac{du^+}{d\xi}\bigg|_{\xi=0} \tag{18}$$

$$\frac{\partial}{\partial x^+}\int_0^{\delta_\theta}(1-\theta)u^+(1+\xi_t)\,d\xi = \frac{1}{Re\,Pr}\frac{d\theta}{d\xi}\bigg|_{\xi=0}. \tag{19}$$

From the boundary conditions of equations (6)–(16), the approximate distributions of the velocity and temperature in the boundary layer can be derived

$$u^+ = (3/2)\eta_v - (1/2)\eta_v^3 \tag{20}$$

$$\theta = (3/2)\eta_\theta - (1/2)\eta_\theta^3. \tag{21}$$

As a first approximation the dimensionless turbulent viscosity is a linear distribution in the boundary layer with ξ as the ordinate

$$\varepsilon_t = (\xi/\delta_v)\xi_{t\infty}. \tag{22}$$

The thickness of the velocity boundary layer can be obtained when the velocity distribution (20) is used in the momentum equation (18) and the solution is

$$\delta_v = A^{-1/2}Re^{-1/2}x^{1/2} \tag{23}$$

where $A = (13/280)+(3/160)\varepsilon_{t\infty}$ and $Re = (\bar{u}\,d_p/v_f)$.

From equations (17) and (22)

$$d\xi = \frac{1}{1+(\xi/\delta_v)\varepsilon_{t\infty}}dy. \tag{24}$$

The relation of the two ordinates can be conducted from the integration of the above equation

$$y^+ = \xi + [\xi^2/(2\delta_v)]\varepsilon_{t\infty}. \tag{25}$$

The thickness of the boundary layer with y as the ordinate is

$$\delta_{vy} = \delta_v(1+0.5\varepsilon_{t\infty}). \tag{26}$$

Since the length of the boundary layer is taken as $L_B^+ = \sqrt{3}l_p^+$ and $l_p^+ = 0.9047(1-\varepsilon_e)^{-1/3}$, $Re = Re_w/\varepsilon_w$, the average thickness of the boundary layer is

$$\bar{\delta}_{vy} = 0.8345(A\,Re_w/\varepsilon_w)^{-1/2}(1-\varepsilon_e)^{-1/6}(1+0.5\varepsilon_{t\infty}). \tag{27}$$

When the temperature distribution (21) and equation (22) are used in the energy equation (19), the differential equation about the ratio of the thickness of the temperature boundary layer η_θ and that of the velocity boundary layer η_v can be derived

$$\gamma^3 + 4\gamma^2 x^+\frac{dr}{dx^+} = \frac{2A}{B\,Pr} \tag{28}$$

where $B = (1/10)+(1/24)\varepsilon_{t\infty}$.

Combined with the boundary conditions, the solution of the above equation is

$$\gamma = (\delta_\theta/\delta_v) = 1.26A^{1/3}B^{-1/3}\,Pr^{-1/3}. \tag{29}$$

So the convective Nusselt number of the heat transfer when the surface is touched by the emulsion phase is

$$Nu_{ecv} = 1.89A^{1/6}B^{1/2}\,Pr^{1/2}\,Re^{1/2}\,x^{r-1/2}. \tag{30}$$

The averaged Nusselt number on the boundary layer with the length of L_B^+ is

$$\overline{Nu}_{ecv} = 1.90A^{1/6}B^{1/3}\,Pr^{1/3}\,(Re_w/\varepsilon_w)^{1/2}(1-\varepsilon_e)^{1/6}. \tag{31}$$

This is the calculating formula for the convective component of the interstitial fluid in the emulsion phase to the heat transfer surfaces.

Determination of the turbulent viscosity in main stream

Galloway and Sage measured the flow field of the interstices in the particles of the packed bed and found that the turbulence intensity $Tu = \sqrt{(\overline{u'^2})}/u$ changed from 0.1 to 0.3 [16]. So Prandtl's turbulent kinetic energy single equation model [17] can be used, in which the turbulent viscosity can be expressed as the product of the root of the turbulent kinetic energy and the dimension of the turbulent length

$$\nu_t = \sqrt{k}l. \tag{32}$$

Assuming the turbulence field is isotropic

$$k = (3/2)Tu^2\,\bar{u}^2. \tag{33}$$

The empirical formula of Escudier [15] can be used for the dimension of the turbulent length

$$l^+ = 0.09\delta_{vy}. \tag{34}$$

The dimensionless turbulent viscosity in the main stream is

$$\varepsilon_{t\infty} = (3/2)^{1/2}Tu\,l^+\left(\frac{Re_w}{\varepsilon_w}\right). \tag{35}$$

From the trial calculation it can be found that the low limit of Galloway and Sage's experimental results $Tu = 0.1$ is adequate.

Determination of the interstitial fluid velocity along the surface of the horizontal tubes

According to Ergun's pressure drop formula in packed beds [16]

$$\nabla p = 150\frac{(1-\varepsilon)^2}{(\phi_s d_p)}\frac{\mu_f u}{\varepsilon^3} + 1.75\frac{(1-\varepsilon)}{\varepsilon^3}\frac{\rho_f u^2}{\phi_s d_p}. \tag{36}$$

This formula combined the velocity with the pressure drop. Since the interstitial fluid velocity is larger than those of the particles and the bubbles in the large particle beds, the above average kinetic energy equations are approximately valid. This equation can be linearized and changed into dimensionless form, as in the treatment of Adams and Welty [13]

$$\nabla p^+ = \left(\frac{\beta_1}{Re} + \beta_2\right)u^+ \tag{37}$$

where

$$p^+ = \frac{p}{\rho_f U_{mf}^2}, \qquad \beta_1 = 150\left[\frac{(1-\varepsilon)}{\varepsilon\phi_s}\right]^2,$$

$$\beta_2 = \frac{1.75}{\varepsilon_\infty}\frac{1-\varepsilon}{\varepsilon\phi_s}, \quad u^+ = u/U_{mf},$$

$$u = U/\varepsilon, \qquad Re = U_{mf}d_p/\nu_f.$$

The polar coordinate is used with the centre of the tube as the origin and the x-axis being perpendicularly down. The changes of the voidage near the tube surfaces are neglected when the pressure field is determined and considered as the velocity distribution is calculated. Combined with the continuing equation, there is

$$\nabla^2 p^+ = 0. \tag{38}$$

The boundary condition are:

at the place far from the tube surfaces

$$\nabla p^+ = \left.\frac{dp^+}{dx^+}\right|_\infty = \frac{1}{\varepsilon_\infty}\left[\frac{\beta_1(\varepsilon_\infty)}{Re} + \beta_2(\varepsilon_\infty)\right]; \tag{39}$$

on the tube surfaces

$$\frac{\partial p^+}{\partial R^+} = 0. \tag{40}$$

Combined with the boundary conditions the solution of equation (38) is

$$p^+ = \left.\left(\frac{\partial p^+}{\partial x^+}\right)\right|_\infty\left(r^+ + \frac{R^{+2}}{r^+}\right)\cos\theta. \tag{41}$$

So the velocity distribution along the tube surface is

$$\left(\frac{\beta_1}{Re} + \beta_2\right)u^+ = \frac{1}{\gamma^+}\left[\frac{\beta_1(\varepsilon_\infty)}{Re} + \beta_2(\varepsilon_\infty)\right]\cdot 2\sin\theta. \tag{42}$$

Considering the voidage of the emulsion phase in the bed bulk is not different obviously with that of the bed at minimum fluidization and it cannot be measured accurately, ε_∞ can be taken as ε_{mf}. So the velocity distribution around the horizontal immersed circular tube is

$$u_w = \frac{U_w}{\varepsilon_w} = \frac{\beta_1(\varepsilon_{mf})/Re + \beta_2(\varepsilon_{mf})}{\beta_1(\varepsilon_w)/Re + \beta_2(\varepsilon_w)}\frac{U_{mf}}{\varepsilon_{mf}}2\sin\theta \tag{43}$$

where $\varepsilon_w = f(\theta)$ is the voidage distribution determined by experiments.

In order to calculate the circumferential averaged heat transfer coefficient around the circular tube, the integral average of u_w can be made

$$\bar{u}_w = \frac{\bar{U}_w}{\bar{\varepsilon}_w} = 2\pi\frac{\beta_1(\varepsilon_{mf})/Re + \beta_2(\varepsilon_{mf})}{\beta_1(\bar{\varepsilon}_w)/Re + \beta_2(\bar{\varepsilon}_w)}\frac{U_{mf}}{\varepsilon_{mf}}. \tag{44}$$

Modelling of the particle conduction component in the emulsion phase

According to the additional assumption 5, the heat transfer of the emulsion phase staying on the immersed surface is divided into convection and conduction regions. The convection component is negligible in the conduction region because the fluid velocity is small. The area of the particle contact point is very small and should be infinitely small in the ideal situation, so it can be neglected. Thus the heat transfer of the particles with the tube surfaces is the conduction through the fluid gaps, which is essentially an unsteady conduction procedure. The thermal diffusion coefficient of the fluid is much larger than that of the solid particles, so the temperature distribution in the fluid gaps is in quasi-steady state and changes with the surface temperature of the particles. According to Fourier's law, the local heat transfer density in the particle conduction regions is

$$\frac{q}{T_B - T_w}\frac{d_p}{k_f} = \frac{\theta_p}{\Delta} = Nu_{ecd}. \quad (45)$$

Thus the key to the problem considered is the determination of the temperature distribution along the particle surfaces. Since the problem is the heat transfer of the tube surface to the spherical particles through the fluid gaps, the whole heat transfer procedure could only be described by the numerical method. Two characteristics were found through the numerical analysis [15]. One is the time averaged Nusselt number changes slightly with time, which indicates that the unsteady effect is not obvious. The other is that the surface temperature of the particles varies very strongly near the contact points and linearly with a large slope, but approaches the bed temperature very fast and does not change with time obviously. So the heat transfer in the conduction region can be simplified. It is assumed that the temperature difference between the particle surface and the immersed surface changes linearly from the contact point to the edge of the conduction region, i.e.

$$\theta = r^+/r^+_{cd}. \quad (46)$$

The dimensionless maximum thickness of the fluid gaps is

$$\Delta_1 = 1/\overline{Nu}_{ecv}. \quad (47)$$

The dimensionless radius of the conduction region r^+_{cd} is

$$r^+_{cd} = r_{cd}/d_p = [\Delta_1(1-\Delta_1)]^{1/2}. \quad (48)$$

Using the temperature distribution in equation (45) and making the area integral average on the conduction region, it can be obtained that

$$\overline{Nu}_{ecd} = \frac{[r^+_{cd}+0.25\sin^{-1}(2r^+_{cd})+r^+_{cd}(0.25-r^{+2}_{cd})^{1/2}]}{r^{+3}_{cd}}. \quad (49)$$

So the heat transfer coefficient of the emulsion packet staying on the immersed surface is

$$Nu_e = (1-f_{cd})\overline{Nu}_{evc} + f_{cd}\,\overline{Nu}_{ecd} \quad (50)$$

where $f_{cd} = (2\pi/\sqrt{3})(r^+_{cd}/l^+_p)^2$ is the area fraction of the conduction region.

HEAT TRANSFER OF BUBBLE PHASE

The bubbles are the isobaric gas mass including a few solid particles. According to Kunii and Levenspiel's analysis to the fluid–solid two-phase flow in the fluidized beds with the potential theory [19] that the inner fluid velocity of a single bubble is $3U_{mf}$ when it stays in the minimum fluidized beds statically. Since the rising velocity of the bubbles is lower than that of the interstitial fluid, the fluid scouring velocity of the bubbles on the heat transfer surfaces can be approximated as $3U_{mf}$. At the same time there are a few particles in the bubbles according to the measurement of the local voidage [15]. The simplified treatment is adopted in which the heat transfer of bubbles is the addition of those of the fluid flow across the horizontal tubes and the particles with the tube surfaces.

For the heat transfer of single phase bubbles with the horizontal tube, Adams and Welty considered that the bubbles only contact the upstream surface, so an analytical solution was derived using the single parameter integral method [13]. Decker and Glickman simplified it to a laminar boundary layer along a plane plate, but they only considered the upstream surface [12]. It can be observed that these treatments are only valid to the situation near the minimum fluidization. When the fluidizing velocity increases, the downstream surface of the horizontal tube is also contacted by bubbles. Since there is no analytical solution to the single phase fluid across the horizontal immersed tube, Churchill and Bernstein's correlation for $Re_{D_T}Pr > 0.2$ [20] should be used

$$Nu_{D_T} = 0.3 + \frac{Re^{1/2}_{D_T}Pr^{1/3}}{[1+(0.4/Pr)^{2/3}]^{1/4}} \times \left[1+\left(\frac{Re_{D_T}}{28\,000}\right)^{5/8}\right]^{4/5}. \quad (51)$$

Considering the fluid flow in the fluidized bed is in turbulence which would strengthen the heat transfer, Dyban and Epik's correlation for $100 \leqslant Re_{D_T}\,Tu \leqslant 10^4$ and' $Tu \leqslant 0.14$ including the influence of the free stream turbulence [21] can be applied

$$Nu_{bcv} = [1+0.09(Re_{D_T}Tu)^{0.2}]\frac{d_p}{D_T}Nu_{D_T}|_{Tu=0} \quad (52)$$

where $Re_{D_T} = 3U_{mf}D_T/\nu_f$ and Tu is taken as the turbulence intensity of the interstitial fluid flow of the emulsion phase.

For the heat transfer of the solid particles in the bubble phase, the conductive effect is considered when they stay at the heat transfer surface and the effect of

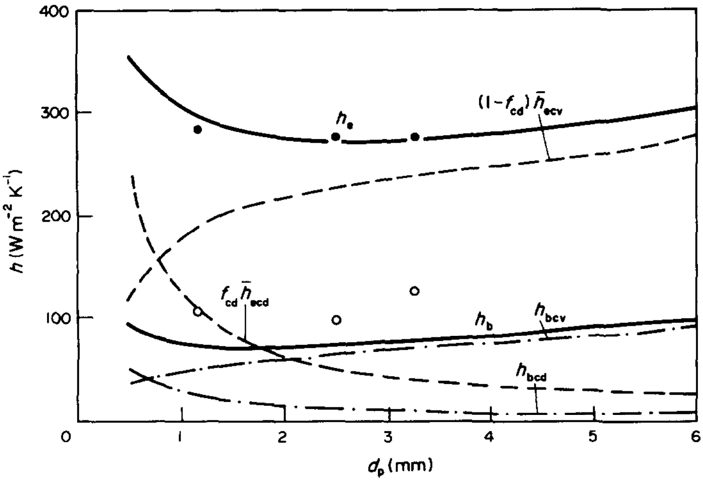

FIG. 3. Variations of heat transfer components with particle dimension: $\phi_s = 1.0$, $\rho_p = 2451$ kg m^{-3}, $\varepsilon_{mf} = 0.40$, $T_B = 20°C$, $Tu = 0.1$, $\varepsilon_e = 0.57$, $\varepsilon_b = 0.93$, $D_T = 40$ mm, $P = 1.03 \times 10^5$ N m^{-2}. ●, maximum; ○, minimum [15].

the fluid flow is omitted. Since the particles in bubbles are scarce and the contact time with the heat transfer surface is much shorter, the heat transfer can be treated as the steady conduction process through an equivalent fluid gap, the thickness of which is that of a fluid cylinder with the diameter d_p and the same volume as the fluid layer between the particle and the tube surface. Considering the voidage of the bubble phase and particle arrangement, the particle conduction component in bubbles is

$$Nu_{bcd} = 1.108(1-\varepsilon_b)^{2/3}. \quad (53)$$

So the heat transfer coefficient when the bubbles contact the horizontal immersed circular tube is

$$Nu_b = Nu_{bcv} + Nu_{bcd}. \quad (54)$$

DISCUSSION

Up to now, each heat transfer component of the immersed surfaces with the fluidized bed at ambient conditions has been modelled and the calculating formulae for the heat transfer coefficient have been derived. If the model is used at high temperatures, the radiative component should be considered, which was given in ref. [15]. In order to use this theoretical model, the parameters needed are the dimension of the particles, sphericity, voidages of the beds at minimum fluidization and the emulsion packet and bubbles near the heat transfer surfaces, physical properties of the fluid, the diameter of the immersed tubes and the contact time fraction of the heat transfer surface with bubbles. The last parameter is given by the empirical correlation [15]

$$f_b = 0.513 \exp[-0.234(U-U_{mf})^{-1}]. \quad (55)$$

Figure 3 gives the variation of each heat transfer component with the change of the particle dimension when the diameter of the horizontal immersed tube is 40 mm. It can be seen that the convective component of the emulsion phase $(1-f_{cd})h_{ecv}$ increases fast at first, then changes slowly with the increasing particle size. This is because the interstitial fluid velocity in the emulsion phase increases with the particle dimension, so the convective component plays a more and more important role. But the change of the conductive component $f_{cd}h_{ecd}$ decreases monotonously. It decreases rather fast when the particles are small, then the rate of change decreases. The additional effect of the convection and conduction makes the heat transfer coefficient of the emulsion phase decrease at first, then does not change obviously in a certain range and increases slightly at the end, which is in good agreement with the experimental results. This indicates the different effects of each component with different particle dimensions. At the same time it is shown in the figure that the convective component of the bubbles h_{bcv} is increased with the particle size and the conductive component changes in the opposite direction. In addition three groups of maximum and minimum values of the circumferential averaged transient heat transfer coefficients were obtained from the experiments [15]. It can be found that the maximum values agree with the theoretical results of the emulsion phase. But the minimum values deviate from the calculated data of the bubble phase. There could be two reasons, the first is the heat inertia of the heat flux probe in the experiments [15], the second may be that modelling of the heat transfer of the bubbles should be improved. Considering this value is not large indeed and would be produced by the time fraction f_b when the total heat transfer coefficients are calculated, the influence would be small.

The theoretical results are compared with the experimental results [15] when the bed materials are

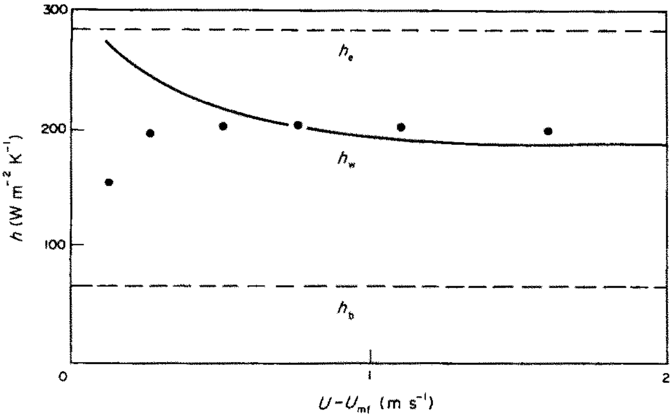

FIG. 4. Comparison of predicted results with experimental data [15]: $d_p = 1.176$ mm, $D_T = 40$ mm. Bed material: sand.

sand with $d_p = 1.176$ mm and glass beads with $d_p = 2.55$ and 3.23 mm in Figs. 4–6, respectively, to check the effectiveness of the proposed model. It can be seen from the comparison that the theoretical results are larger than the experimental data near the minimum fluidizing velocity. When $U-U_{mf} > 0.4$ m s^{-1}, the two kinds of results are in good agreement. So it can be concluded that the theoretical model proposed can be used to predict the heat transfer of the immersed surface with the large particle fluidized beds at the normal fluidizing state. It reflects the main characteristics of the heat transfer at this kind of operation conditions. Though some simplifications are made in order to carry out the modelling and obtain the simple calculating formulae, this model has included the factors playing dominate roles and can be used in practical engineering calculations.

Figure 7 gives the comparison of the predicted results of the theoretical models proposed by different workers. It can be found that the model proposed in this paper can model the heat transfer process of the immersed tubes with the beds in a rather wide particle dimension range. The predicted results of Catipovic *et al.*'s model [9] are obviously lower than the experimental data [15, 22]. The heat transfer coefficients calculated with Zabrodsky *et al.*'s model [10] changes strongly with the particle size. When the particle dimension is in smaller and larger ranges, the theoretical results agree with the experimental values, but the differences are rather large in the range of d_p = 1–3 mm. Ganzha *et al.*'s model [11] is closer to the experimental results than that of Catipovic *et al.* [9], but is still lower. Decker and Glickman's model [12] agrees with experimental results at $d_p = 2$ mm, but the agreement is decreased at other particle dimensions. So it can be concluded that the theoretical model in this paper agrees with experimental results more when compared with the models of other workers.

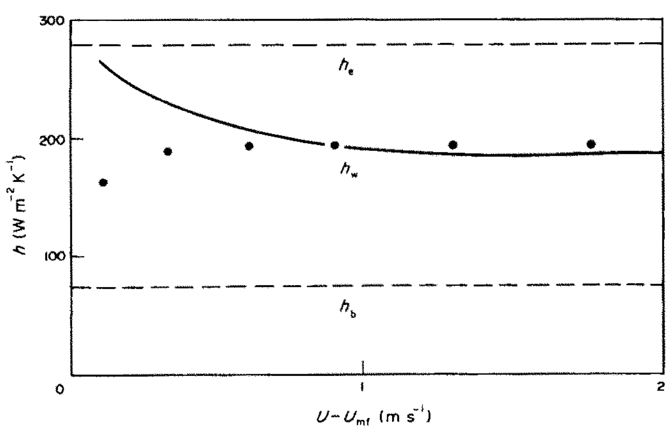

FIG. 5. Comparison of predicted results with experimental data [15]: $d_p = 2.55$ mm, $D_T = 40$ mm. Bed material: glass beads.

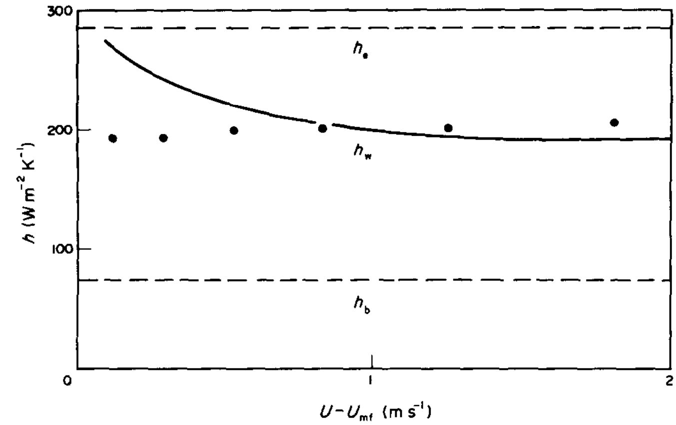

FIG. 6. Comparison of predicted results with experimental data [15]: $d_p = 3.23$ mm, $D_T = 40$ mm. Bed material: glass beads.

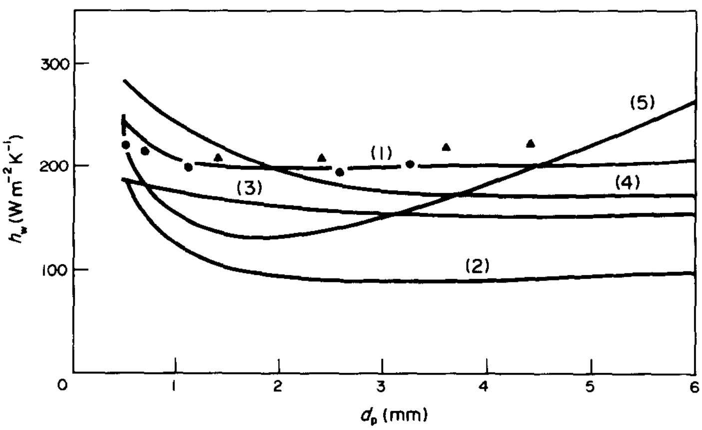

FIG. 7. Comparison of predicted results of different workers. Calculating conditions: same as in Fig. 3, $U-U_{mf} = 1.0$ m s^{-1}. (1) Present paper. (2) Catipovic *et al.* [9]. (3) Zabrodsky *et al.* [10]. (4) Ganzha *et al.* [11]. (5) Decker and Glickman [12].

REFERENCES

1. H. S. Mickley and D. F. Fairbanks, Mechanism of heat transfer to fluidized beds, *A.I.Ch.E. Jl* **1**, 374–384 (1955).
2. A. P. Baskakov, The mechanism of heat transfer between a fluidized bed and a surface, *Int. Chem. Engng* **4**, 320–324 (1964).
3. A. G. Gorelik, *J. Engng Phys.* **13**, 495–498 (1967).
4. J. Kubie and J. Broughton, A model of heat transfer in gas fluidized beds, *Int. J. Heat Mass Transfer* **18**, 651–654 (1971).
5. J. S. M. Botterill and J. R. Williams, The mechanisms of heat transfer gas-fluidized beds, *Trans. Instn Chem. Engrs* **41**, 217–230 (1963).
6. J. S. M. Botterill, M. H. D. Butt, G. L. Cain and K. A. Redish, *Int. Symp. on Fluidization*, Drinkenberg, pp. 442–457. Netherland University Press, Amsterdam (1967).
7. R. A. A. Brusenback, Microscopic analysis of fluidized bed heat transfer, Ph.D. thesis, Northwestern University (1963).
8. J. D. Gabor, Wall-to-bed heat transfer in fluidized bed and packed bed, *Chem. Engng Symp. Series* **66**(105), 76–86 (1970).
9. N. M. Catipovic, G. N. Jovanovic, T. J. Fitzgerald and O. A. Levenspiel, A model for heat transfer to horizontal tubes immersed in a fluidized bed of large particles. In *Fluidization* (Edited by J. R. Grace and J. M. Matsen), pp. 225–234. Plenum Press, New York (1980).
10. S. S. Zabrodsky, Yu. G. Epanov, D. M. Galershtein, S. C. Saxena and A. K. Kolar, Heat transfer in a large particle fluidized bed with immersed in-line and staggered bundles of horizontal smooth tubes, *Int. J. Heat Mass Transfer* **24**, 571–579 (1981).
11. V. L. Ganzha, S. N. Upadhyay and S. C. A. Saxena, Mechanistic theory for heat transfer between fluidized beds of large particles and immersed surface, *Int. J. Heat Mass Transfer* **25**, 1531–1540 (1982).
12. N. A. Decker and L. R. Glickman, Heat transfer in large particle fluidized beds, *Int. J. Heat Mass Transfer* **26**, 1307–1320 (1983).
13. R. L. Adams and J. R. Welty, A gas convective model of heat transfer in large particle fluidized beds, *A.I.Ch.E. Jl* **25**, 395–450 (1979).
14. D. Geldart, Type of gas fluidization, *Powder Technol.* **7**, 285–292 (1973).
15. J. D. Lu, Theoretical and experimental studies to heat

transfer between horizontal immersed tubes and fluidized beds, Ph.D. thesis, Huazhong University of Science and Technology (1988).
16. T. R. Galloway and B. H. A. Sage, A model of the mechanism of transfer in packed, distended and fluidized beds, *Chem. Engng Sci.* **25**, 495–516 (1970).
17. B. E. Launder and D. B. Spalding, *Mathematical Models of Turbulence*. Academic Press, New York (1972).
18. S. Ergan, Fluid flow through packed column, *Chem. Engng Prog.* 48 (1952).
19. O. Kunii and O. Levenspiel, *Fluidization Engineering*. Wiley, New York (1969).
20. S. W. Churchill and M. Bernstein, A correlation equation for forced convection from gases and liquids to a circular cylinder in cross flow, *J. Heat Transfer* **99**, 300–306 (1977).
21. E. P. Dyban and E. Ya. Epik, *Heat and Mass Transfer (Teplo-massoperenos)*, *Trans. IVth All-Union Cong. on Heat and Mass Transfer*, Minsk, Vol. 1, p. IV (1972).
22. J. Hu, T. Z. Ma, Z. F. Zhang, J. Q. Zhao and X. H. Huo, Studies on local heat transfer around a horizontal tube in a large particle fluidized bed, *J. Engng Thermophys.* **8**, 255–258 (1987), in Chinese.

MODELISATION DU TRANSFERT THERMIQUE ENTRE DES SURFACES IMMERGEES ET DES LITS FLUIDISES A GROSSES PARTICULES

Résumé—Un nouveau modèle théorique est proposé pour le transfert thermique entre des surfaces immergées et des lits fluidisés à grosses particules. Le transfert de chaleur de la phase émulsion à la surface immergée est traité comme la somme de la part convective de l'écoulement du fluide intersticiel et de la part conductive des particules solides. Le transfert des bulles à la surface est aussi considéré. Les formules théoriques obtenues sont en très bon accord avec les données expérimentales dans un large domaine.

THEORETISCHE BETRACHTUNG DES WÄRMEÜBERGANGS AN EINER OBERFLÄCHE IN EINEM WIRBELBETT AUS GROBKÖRNIGEN PARTIKELN

Zusammenfassung—Es wird ein neues Modell zur Berechnung des Wärmeübergangs zwischen einer in ein Wirbelbett mit grobkörnigen Partikeln eingetauchten Oberfläche und dem umgebenden Fluid vorgeschlagen. Der Wärmeübergang setzt sich additiv aus zwei Anteilen zusammen: aus der Konvektion der zwischen den Partikeln auftretenden Strömung und der Wärmeleitung der Festkörperpartikel. Der Wärmeübergang zwischen Blasen und der Oberfläche wird ebenfalls betrachtet. In einem großen Bereich stimmen die experimentellen Daten mit den berechneten Werten überein.

МОДЕЛЬНОЕ ИССЛЕДОВАНИЕ ТЕПЛООБМЕНА МЕЖДУ ПОГРУЖЕННЫМИ ПОВЕРХНОСТЯМИ И ПСЕВДООЖИЖЕННЫМИ СЛОЯМИ КРУПНЫХ ЧАСТИЦ

Аннотация—Предложена новая теоретическая модель для описания теплообмена между погруженными поверхностями и псевдоожиженными слоями крупных частиц. Перенос тепла от псевдоожиженного слоя к поверхности рассматривается как сумма конвективной составляющей, обусловленной потоком газовой фазы, и кондуктивной—через частицы. Теплообмен между пузырями и поверхностью также принимается во внимание. Полученные теоретические расчетные формулы хорошо согласуются с экспериментальными данными.